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APUNTS

TÍTOL / **TRANSMISSION LINES AND RADIOFREQUENCY CIRCUITS ; ANTENNAS**

AUTOR / Collado, Juan Carlos; González, Jose María

ASSIGNATURA / Aeronautical Communications

MATÈRIA / Enginyeria Aeronàutica

TITULACIÓ / Grau en Enginyeria d'aeronavegació

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1. TRANSMISSION LINES AND RADIOFREQUENCY CIRCUITS

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    - low-loss lines
  - Reflection coefficient → forward and reverse propagating waves
  - Power and losses: return loss → attenuation
  - Voltage standing wave ratio
  - Impedance → impedance of transmission lines
  - Generator mismatch

[COLLIN] R.E. Collin, *Foundations for Microwave Engineering*, Wiley-Interscience, 2nd Edition, 2001 (New York)
[POZAR] D.M. Pozar, *Microwave Engineering*, Addison-Wesley Publishing Company, 2nd Edition, 1993 (Reading, Massachusetts)

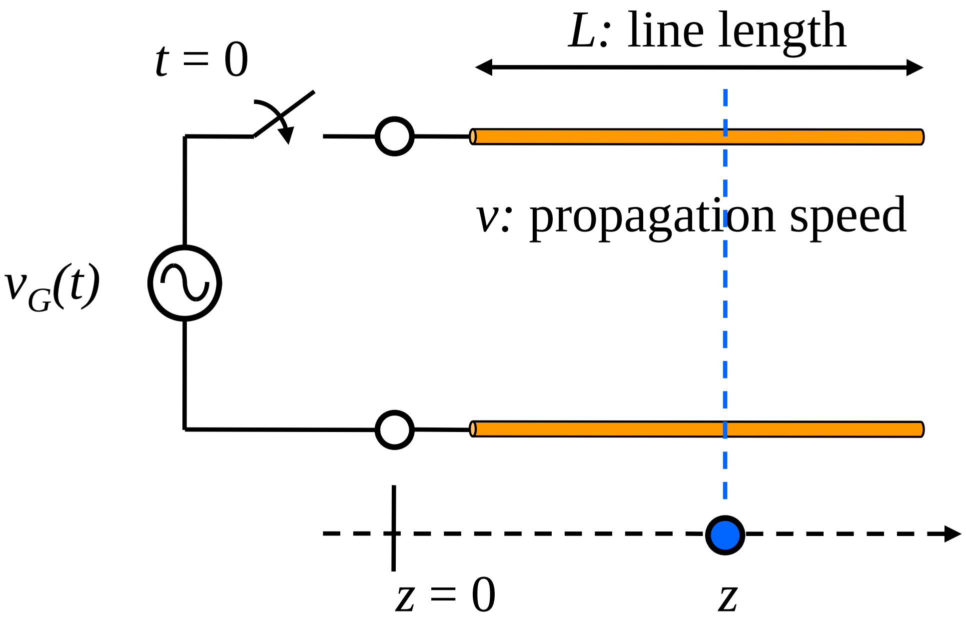
GLOSSARY

- α : attenuation constant [m^{-1}]
- β : phase constant [$rad \cdot m^{-1}$]
- C_d : distributed capacitance per unit length [F/m]
- *CNS* : Communications, Navigation, Surveillance
- ε_0 : electric permittivity of vacuum [$8.85 \cdot 10^{-12}$ F/m]
- f_0 : frequency [Hz]
- γ : propagation constant [m^{-1}]
- G : distributed conductance per unit length [S/m]
- $i(z,t)$: current in time domain [V]
- I_0^+ : current amplitude of progressive wave at z=0 [A]
- l : transmission line length [m]
- L_d : distributed inductance per unit length [H/m]
- λ : wavelength [m]
- μ_0 : magnetic permeability of vacuum [$4\pi \cdot 10^{-7}$ H/m]
- ω : angular frequency [rad/s]
- R : distributed resistance per unit length [Ω/m]
- *RF* : radiofrequency
- T : period [s]

GLOSSARY

- R_S : surface resistivity [Ω/square]
- δ_S : skin depth [m]
- σ : conductivity [Sm]
- σ_d : dielectric conductivity [Sm]
- $tan\ \delta$: loss tangent [adim]
- RL : return loss [dB]
- ρ : (voltage) reflection coefficient [adim]
- ρ_G : (voltage) generator reflection coefficient [adim]
- ρ_{IN} : (voltage) reflection coefficient at input port [adim]
- ρ_L : (voltage) load reflection coefficient [adim]
- $v(z,t)$: voltage in time domain [V]
- V_G : voltage at generator [V]
- V_0^+ : voltage amplitude of progressive wave at z=0 [V]
- v_p : phase velocity [m/s]
- $VSWR$: Voltage Standing Wave Ratio [adim]
- Z_G : generator impedance [Ω]
- Z_{IN} : impedance at the input port of the transmission line [Ω]
- Z_L : load impedance [Ω]
- Z_0 : transmission line characteristic impedance [Ω]

- Wave propagation along a transmission line considering the propagation delays:



Source voltage:

$$v_G(t) = A \cdot \cos(\omega_0 t)$$

propagation delay

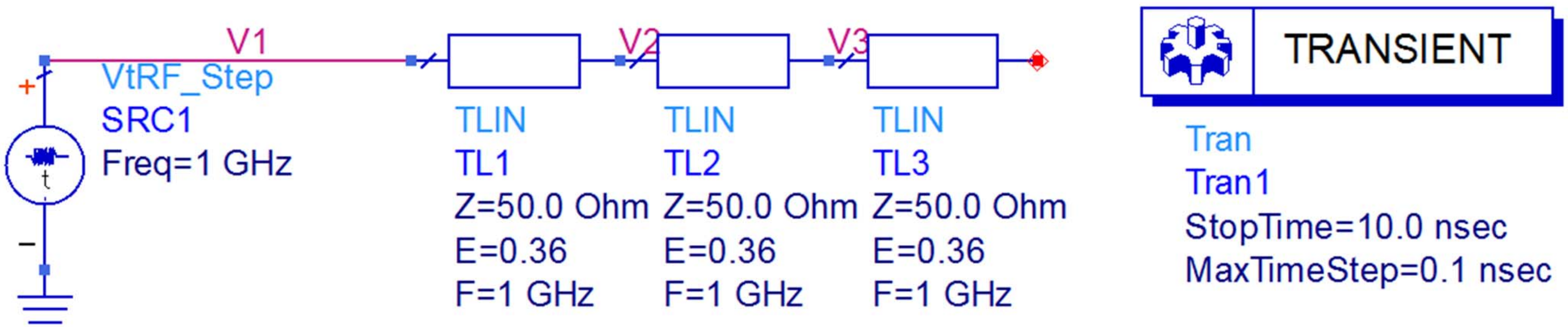
Voltage:

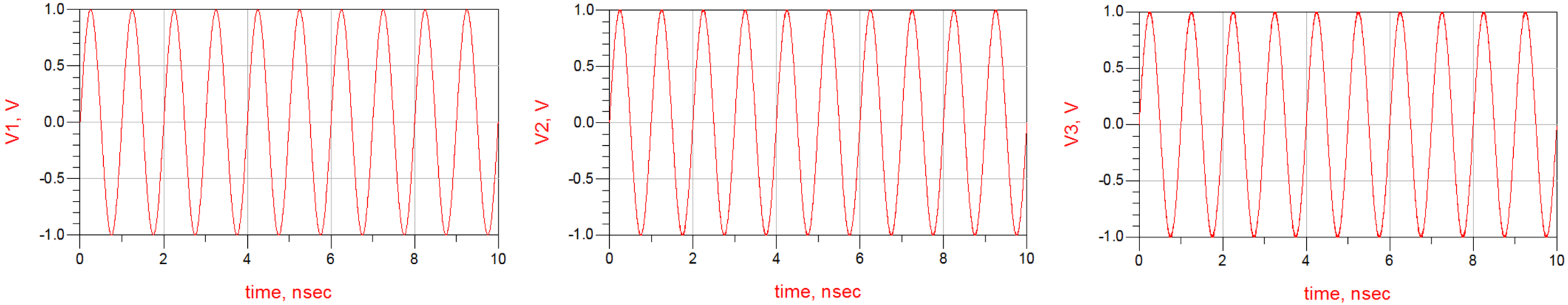
$$v(z,t) = A \cdot \cos\left(\omega_0 \left[t - \frac{z}{v}\right]\right)$$

- Some remarks:
  - Wave propagation considers the propagation delays.
  - Each point in the line has a different voltage/current at the same time t.
  - Periodicity in time or *period* T.
  - Spatial periodicity or *wavelength* λ.
  - Dimensions use to be define with respect to λ.

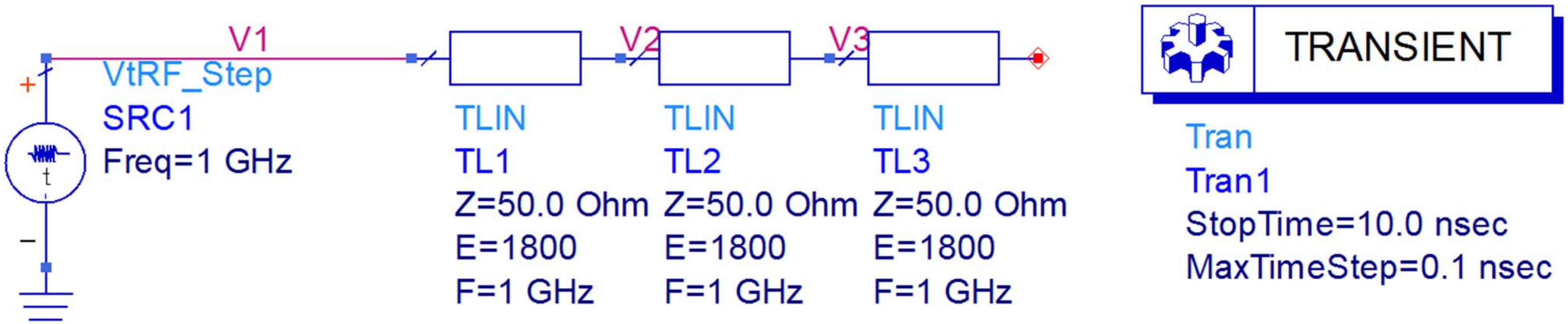
$$T = \frac{2\pi}{\omega_0} \qquad \lambda = \frac{2\pi}{\omega_0 / v} = \frac{v}{f_0}$$

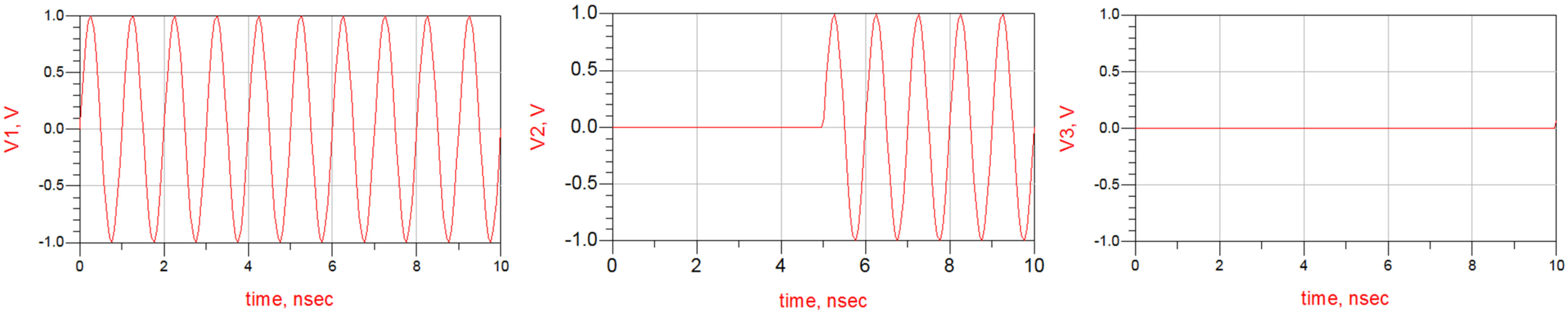
Example: Propagation at Low Frequencies. Consider a circuit having a transmission line length of 0.003λ (3 lines, 0.001λ-length each) fed with a sinusoidal wave of 1 GHz.





Example: Propagation at High Frequencies. Consider a circuit having a transmission line length of 15λ (3 lines, 5λ-length each) fed with a sinusoidal wave of 1 GHz.



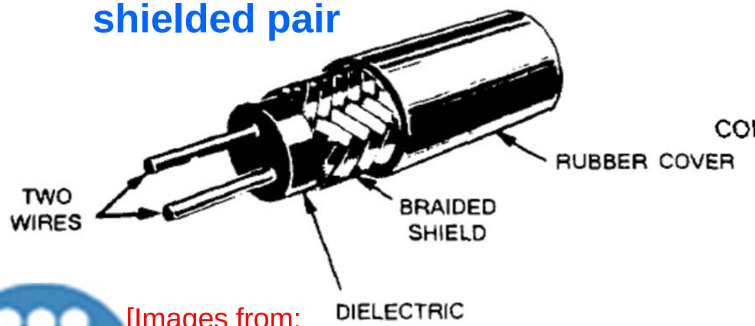


- Transmission line: a set of two or more than two conductors in a dielectric medium having a uniform transversal section that allow the propagation of a EM wave.
- Transmission lines are physical devices whose purpose is to guide electromagnetic waves (carry RF power) from one place to another.
- They are capable of guiding TEM waves (TEM waves can only exist in structures containing two or more separated conductors).
- Two-wire transmission lines are inefficient for transfering electromagnetic energy at high frequencies due to the lack of confinement in all directions.
- Coaxials are more efficient than two-wire lines in those cases.

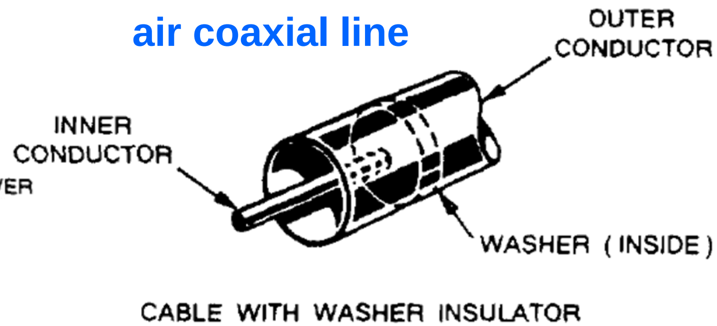
two-wire ribbon line (twin lead)

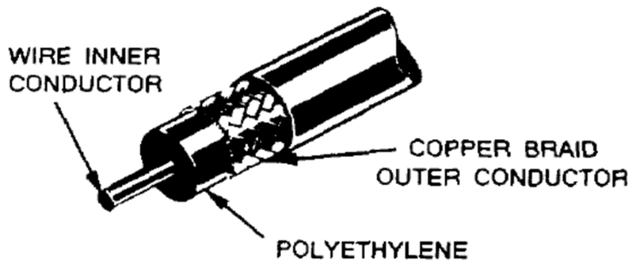
twisted pair

shielded pair



air coaxial line

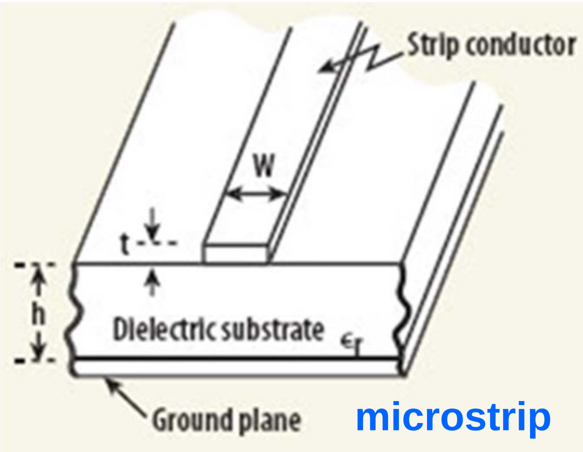




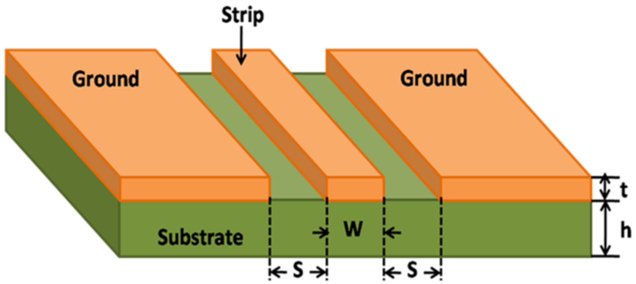
flexible coaxial line

[Images from: http://www.techlearner.com/Apps/TransandGuides.pdf]

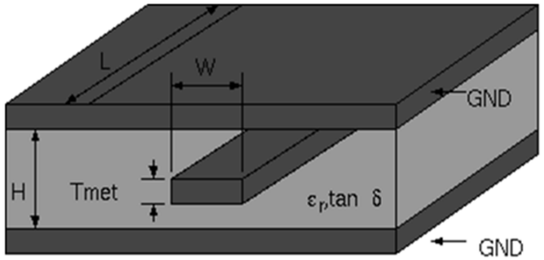
- There are lots of planar structures used as transmission lines. Metallic parts are supported by dielectrics (fiberglass, ceramics, foams,...).



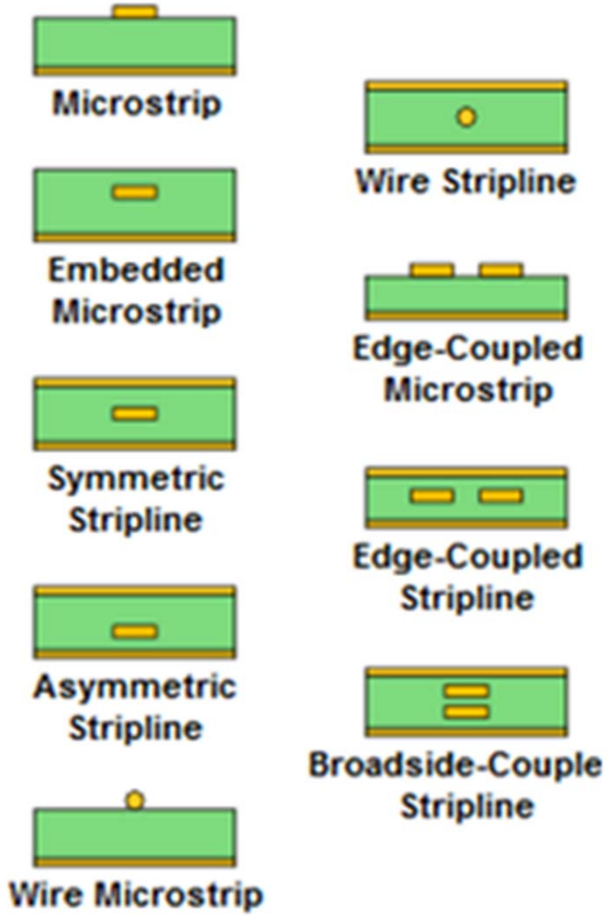
microstrip



coplanar transmission line

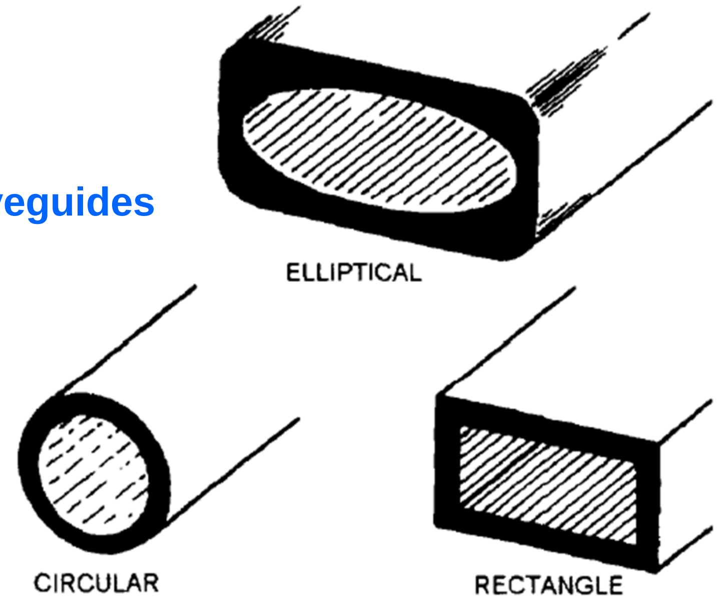


stripline



- Waveguides are the most efficient. They are fabricated with just one conductor. Waveguides do not support TEM waves.
- Two-wire lines are less bulky and less expensive than waveguides.

waveguides

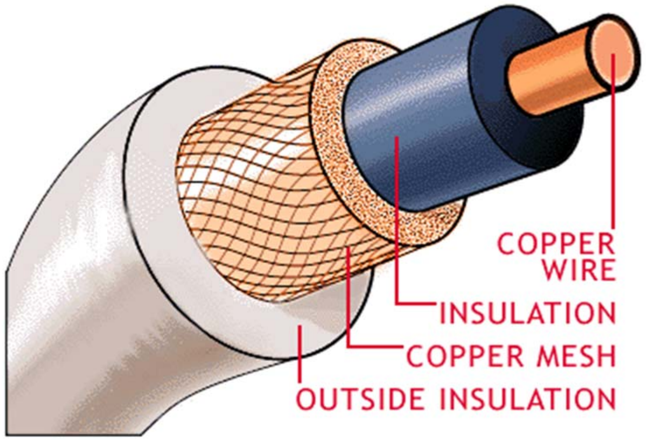


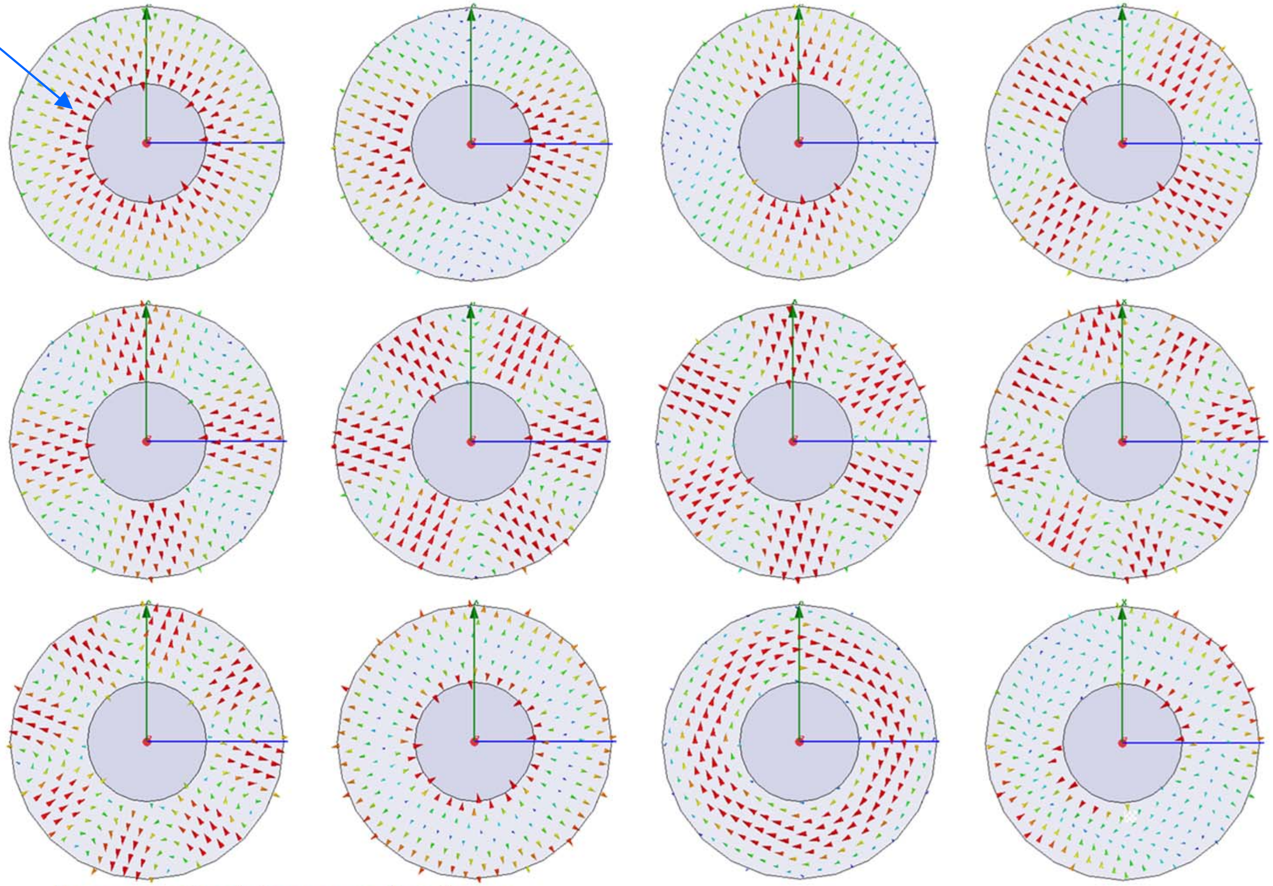
[Image from: http://www.techlearner.com/Apps/TransandGuides.pdf]

- The equations that rule the propagation of electromagnetic waves (Maxwell's equations) and the geometry of a given structure bound the electromagnetic field configurations that supports.
- These configurations are called (field) **modes**.

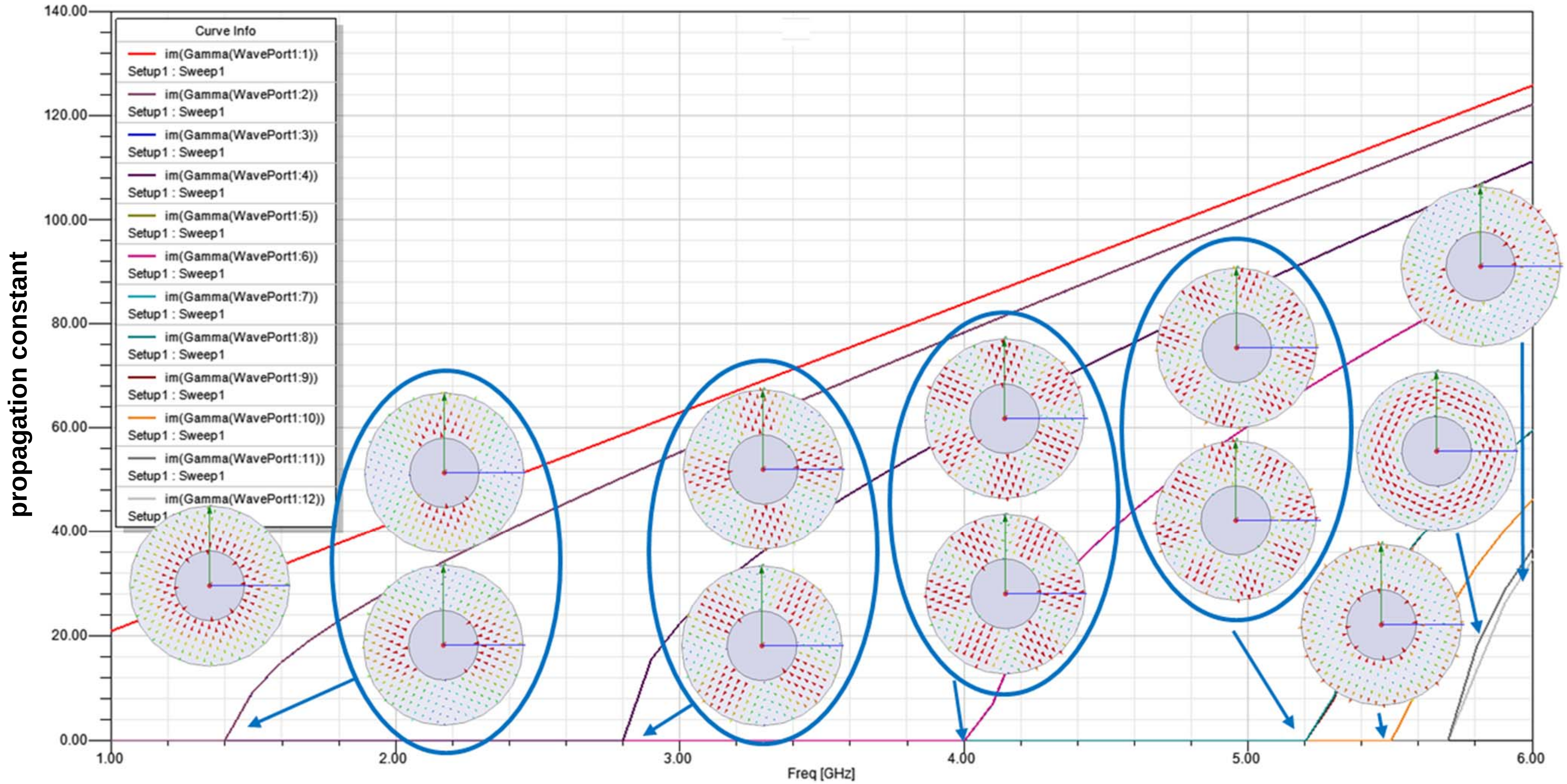
first modes inside a coaxial cable

[Image from: http://www.phy.davidson.edu/stuhome/phstewart/IL/speed/Cableinfo.html]

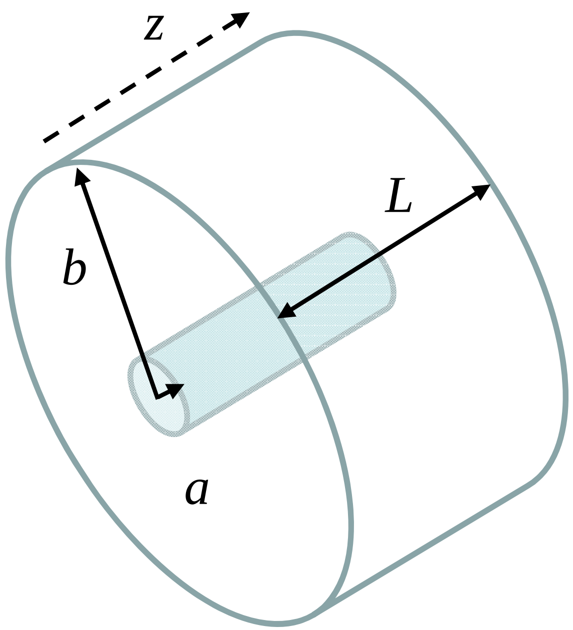




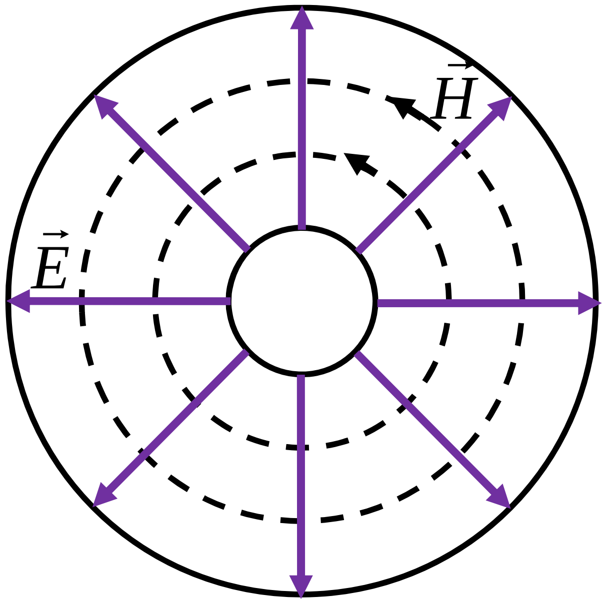
- The boundary conditions posed by the structures and set-ups (materials, voltage and/or current sources, frequencies) make possible that these modes do propagate or not inside the structures.



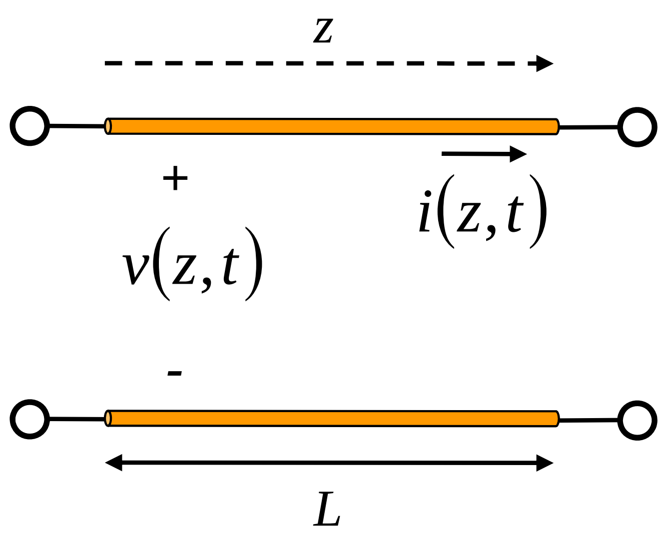
- A coaxial transmission line supports a TEM mode (electric field orientation, magnetic field orientation, and energy propagation direction form a triad).



physical structure



electromagnetic field distribution
(TEM mode)

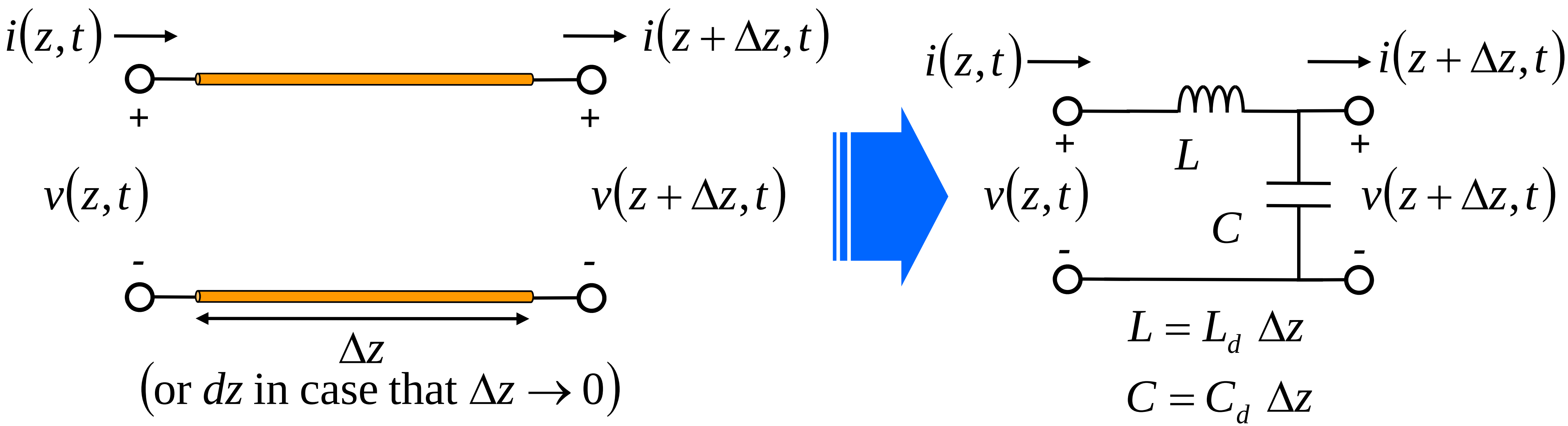


abstract model
(ideal transmission line)

$$v(z,t) = -\int \vec{E}(z,t)\, d\vec{l}$$

$$i(z,t) = \oint \vec{H}(z,t)\, d\vec{l}$$

- The knowledge of voltage and current waves propagating along the transmission line allows the use of a distributed circuit model to analyze its performance.
- The model represents an infinitesimally short segment of the transmission line
- This model is convenient to explore properties of lines without knowing the fields in detail. However, the structures should be analyzed in detail if accurate performances have to be known.

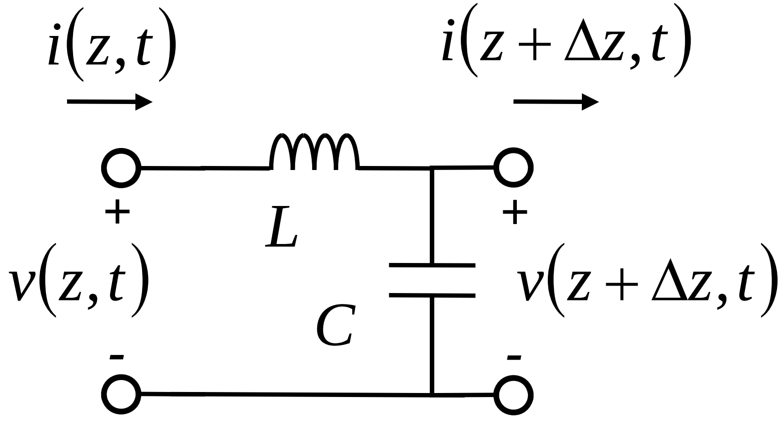


- L_d and C_d are the distributed inductance [H/m] and capacitance [F/m] associated to the coaxial structure and materials. **No losses are assumed** in this example (meaning that there is no distributed resistance).

- Applying Kirchhoff's voltage and current laws:

$$v(z,t) - L_d \Delta z \frac{\partial i(z,t)}{\partial t} = v(z+\Delta z,t)$$

$$i(z,t) - C_d \Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} = i(z+\Delta z,t)$$



$$L = L_d\ \Delta z$$

$$C = C_d\ \Delta z$$

- Dividing by Δz and taking the limit $\Delta z \rightarrow 0$:

$$\frac{\partial v(z,t)}{\partial z} = -L_d \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -C_d \frac{\partial v(z,t)}{\partial t}$$

- Considering sinusoidal **steady-state condition** (cosine based phasors) (**TRANSIENTS NOT CONSIDERED**):

$$\frac{\partial V}{\partial z} = -j\omega L_d I$$

$$\frac{\partial I}{\partial z} = -j\omega C_d V$$

- The **wave equations** can be solved simultaneously:

$$\frac{\partial^2 V}{\partial z^2} = -\beta^2 V$$

$$\frac{\partial^2 I}{\partial z^2} = -\beta^2 I$$

being: $\beta = \omega\sqrt{L_d C_d}$

phase constant

- Solutions are:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

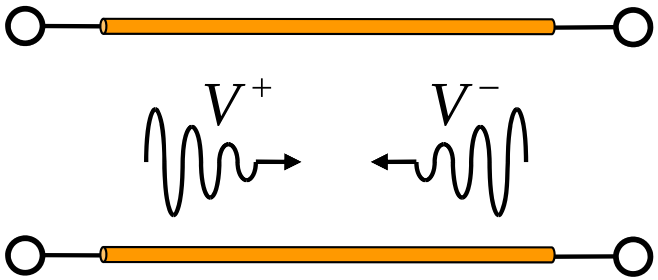
$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$

being: $I_0^+ = \dfrac{V_0^+}{Z_0}$ $\quad I_0^- = -\dfrac{V_0^-}{Z_0}$

and: $Z_0 = \sqrt{\dfrac{L_d}{C_d}}$

characteristic impedance

- Meaning that there are waves propagating in opposite directions along the transmission line (positive and negative waves).



- Wave propagation solutions in time domain are:

$$v(z,t)=\left|V_0^+\right|\cos\left(\omega t-\beta z+\arg\left[V_0^+\right]\right)+\left|V_0^-\right|\cos\left(\omega t+\beta z+\arg\left[V_0^-\right]\right)$$

$$i(z,t)=\left|I_0^+\right|\cos\left(\omega t-\beta z+\arg\left[I_0^+\right]\right)+\left|I_0^-\right|\cos\left(\omega t+\beta z+\arg\left[I_0^-\right]\right)$$

the wave propagation on the line means a delay (in fact a phase delay) ⇒ **lengths *l* are given in terms of λ or in degrees (βl)**

- Wavelength and phase velocity on the line are:

$$\lambda=\frac{2\pi}{\beta} \qquad v_p=\frac{\omega}{\beta}=\lambda f=\frac{1}{\sqrt{L_d C_d}}$$

v_p does not change with frequency: NO DISTORTION (each frequency component of a signal travels at the same v_p along the line)

- Do not forget… that the electrical model parameters **depend** on line **geometry**.

physical structure $\vec{E}$ $\vec{H}$ ⇒ **distributed model parameters** L_d C_d ⇒ **electrical model parameters** β Z_0

Take your time…

Characteristic impedance. Given the solutions of the voltage and current wave equations derived for a two-wire lossless transmission line differential equations:

$$\frac{\partial V}{\partial z} = -j\omega L_d I \qquad V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\frac{\partial I}{\partial z} = -j\omega C_d V \qquad I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$

being: $\beta = \omega\sqrt{L_d C_d}$

Find the characteristic impedance of the line and the relation between the voltage and current waves.

Take your time…

Solution: Characteristic impedance.

Substituting the solution for the current into the differential equation for the voltage:

$$\frac{\partial V}{\partial z} = -j\omega L_d I$$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$$

$$-j\beta V_0^+ e^{-j\beta z} + j\beta V_0^- e^{j\beta z} = -j\omega L_d \left(I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}\right)$$

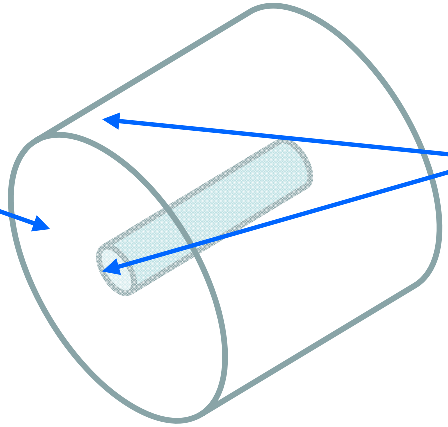
$$I_0^+ = \frac{V_0^+}{\omega L_d}\beta = \frac{V_0^+}{\sqrt{\dfrac{L_d}{C_d}}} = \frac{V_0^+}{Z_0}$$

characteristic impedance

$$I_0^- = -\frac{V_0^-}{Z_0}$$

Take your time…

Lossy two-wire transmission line. In previous slides we presented the model of a two-wire lossless transmission line. Suggest a model for a lossy two-wire transmission line (e.g. lossy coaxial cable).



Hints:

Losses in metals are characterized by their **electrical conductivity** σ (units: Ω^{-1}m=Sm) or by their **surface resistivity** R_S (units: Ω/square). R_S represents the mean power absorbed by a unit area (1 m^2) and can be used on planar surfaces or surfaces with curvature radius smaller than the **skin depth** δ_S. Currents on metals flow mainly in their outer "skin" an a level called skin depth.

$$R_S = \frac{1}{\delta_S \sigma}$$

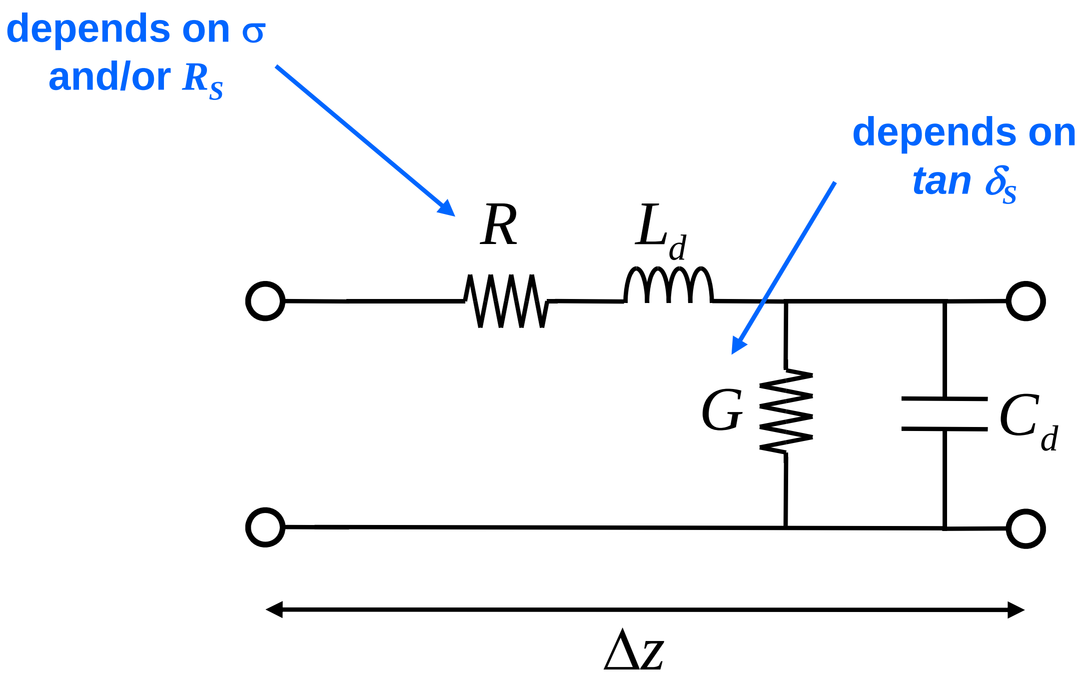
$$\delta_S = \frac{1}{\sqrt{\pi \mu \sigma f}}$$

Losses in dielectrics are characterized by their **loss tangent** *tan* δ. It represents the tangent of the angle in the complex plane between the electric field resistive losses and its lossless reactive component.

$$\tan \delta = \frac{\omega \varepsilon'' + \sigma_d}{\omega \varepsilon'}$$

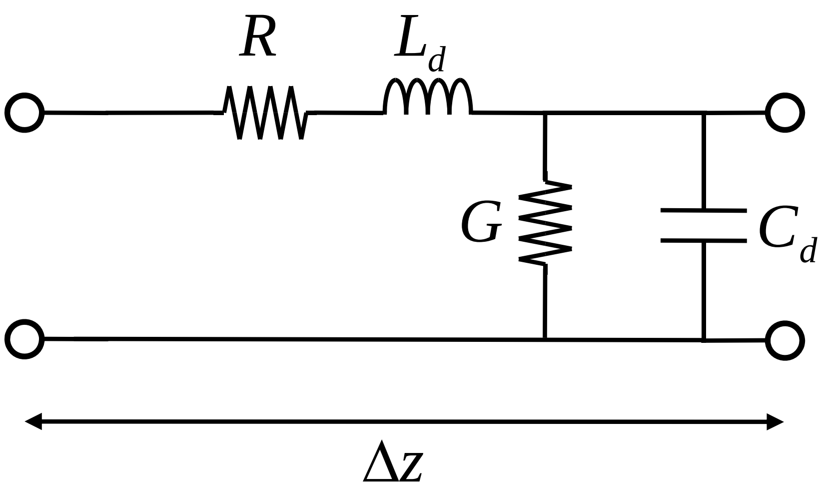
Take your time…

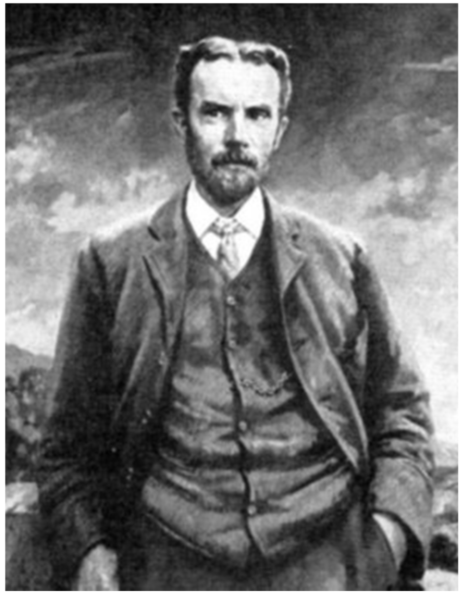
Solution. Lossy two-wire transmission line.



Take your time…

Telegrapher's equations. Derive the propagation equations for a lossy two-wire transmission line having the lumped-circuit model of the figure. The resulting set of differential equations are called the **telegrapher's equations** and are due **Oliver Heaviside** in 1880.





[Image from: http://es.wikipedia.org/wiki/Oliver_Heaviside]

L_d distributed inductance per unit length [H/m]
C_d distributed capacitance per unit length [F/m]
R distibuted resistance per unit length, for both conductors [Ω/m]
G distibuted conductance per unit length, for both conductors [S/m]

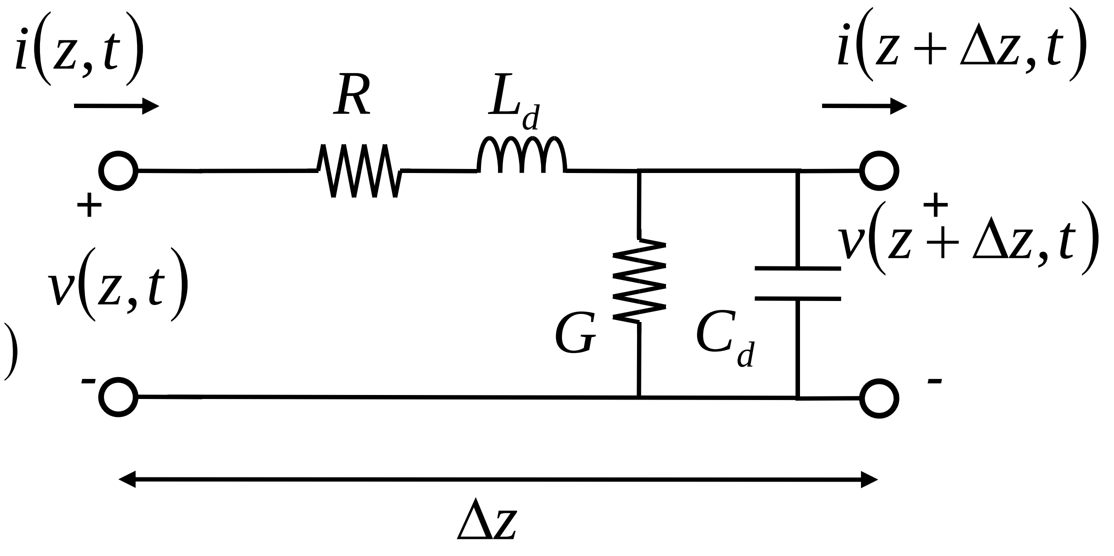
Take your time…

Solution: Telegrapher’s equations.

- According to Kirchhoff’s laws:

$$v(z,t) - L_d \Delta z \frac{\partial i(z,t)}{\partial t} - R\Delta z\, i(z,t) = v(z+\Delta z,t)$$

$$i(z,t) - G\Delta z\, v(z+\Delta z,t) - C_d \Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} = i(z+\Delta z,t)$$



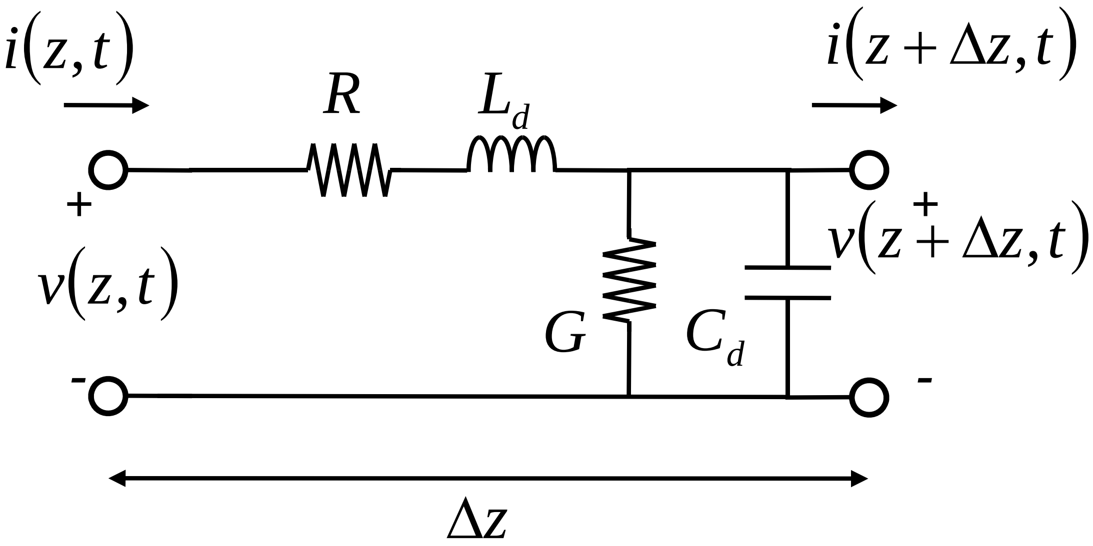
- Dividing by Δz and taking the limit $\Delta z \to 0$:

$$\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L_d \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C_d \frac{\partial v(z,t)}{\partial t}$$

Telegrapher’s equations

- From the lumped-circuit model of a **lossy** two-wire transmission line we get the telegrapher's equations.



$$\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L_d \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C_d \frac{\partial v(z,t)}{\partial t}$$

- Considering sinusoidal steady-state condition (cosine based phasors):

$$\frac{\partial V}{\partial z} = -(R + j\omega L_d) I$$

$$\frac{\partial I}{\partial z} = -(G + j\omega C_d) V$$

- Wave equations are now:

$$\frac{\partial^2 V}{\partial z^2} = \gamma^2 V$$

$$\frac{\partial^2 I}{\partial z^2} = \gamma^2 I$$

being: $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L_d)(G + j\omega C_d)}$

complex propagation constant

attenuation

phase constant

v_p depends with frequency: DISTORTION (each frequency component of a signal travels at different v_p along the line)

- The solutions are:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right)$$

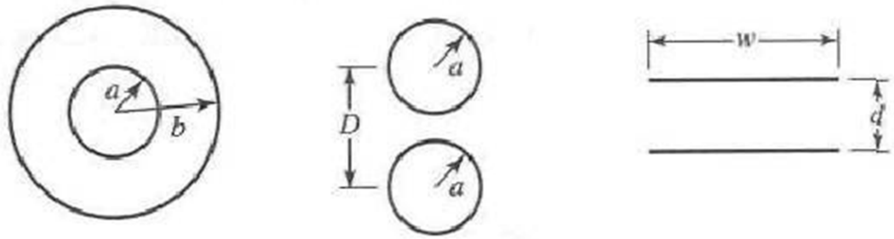
being: $Z_0 = \sqrt{\dfrac{R + j\omega L_d}{G + j\omega C_d}}$

- And in time domain:

$$v(z,t) = \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \arg\left[V_0^+\right]\right) + \left|V_0^-\right| e^{\alpha z} \cos\left(\omega t + \beta z + \arg\left[V_0^-\right]\right)$$

- Lumped-circuit parameters required to model some common lines as a function of their dimensions, surface resistivity (R_S), and materials filling the space between the conductors (permitivitty $\varepsilon = \varepsilon' - j\varepsilon''$ and permeability $\mu = \mu_0 \mu_r$).

TABLE 2.1 Transmission Line Parameters for Some Common Lines



	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

[Table from: [POZAR]]

parameters for other transmission lines are found in the literature or by using specific software

- In practical lines losses are small. The equations for attenuation and propagation factor can be simplified.
  - The complex propagation factor can be re-arranged:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L_d)(G + j\omega C_d)} = j\omega\sqrt{L_d C_d}\sqrt{1 - j\left(\frac{R}{\omega L_d} + \frac{G}{\omega C_d}\right) - \frac{RG}{\omega^2 L_d C_d}}$$

  - For a low-loss line we can assume: $R << \omega L_d$ and $G << \omega C_d$

$$\gamma \approx j\omega\sqrt{L_d C_d}\sqrt{1 - j\left(\frac{R}{\omega L_d} + \frac{G}{\omega C_d}\right)} \approx j\omega\sqrt{L_d C_d}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L_d} + \frac{G}{\omega C_d}\right)\right]$$

$$Z_0 = \sqrt{\frac{R + j\omega L_d}{G + j\omega C_d}} \approx \sqrt{\frac{L_d}{C_d}}$$

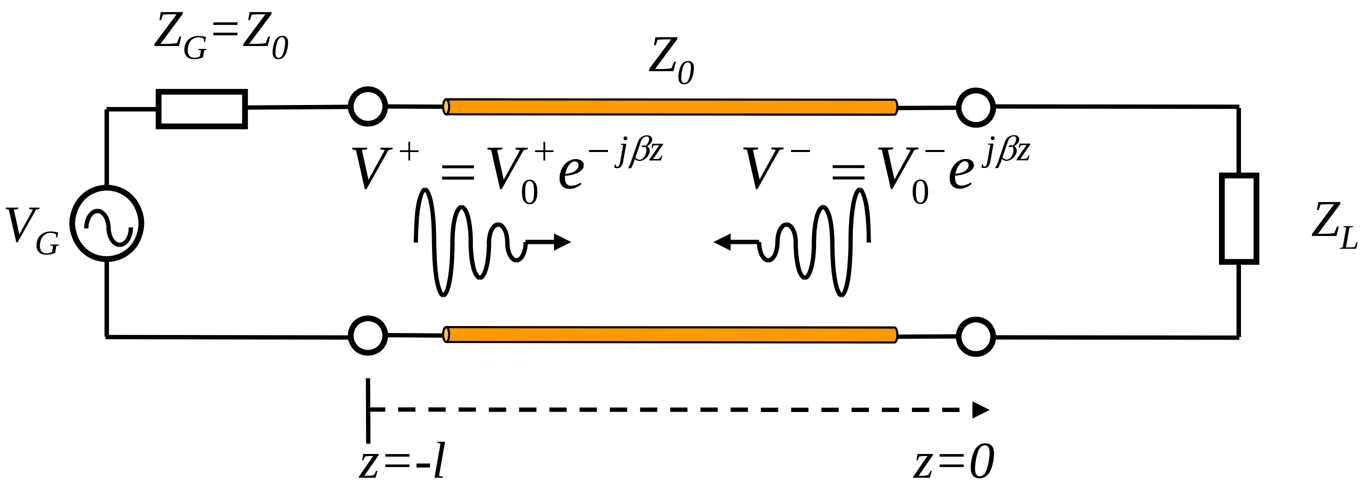
same values than lossless lines

  - Finally:

$$\alpha \approx \frac{1}{2}\left(R\sqrt{\frac{C_d}{L_d}} + G\sqrt{\frac{L_d}{C_d}}\right) \qquad \beta \approx \omega\sqrt{L_d C_d} \qquad Z_0 \approx \sqrt{\frac{L_d}{C_d}}$$

DISTORTIONLESS

- Wave reflection on a transmission line can be illustrated by considering a lossless transmission line loaded with an arbitrary impedance Z_L. Z_0 is the characteristic impedance of the transmission line.



- A voltage reflection coefficient can be defined for any point in the line as the amplitude of the reflected voltage wave normalized to the amplitude of the incident voltage wave.

$$\rho(z) = \frac{V^-(z)}{V^+(z)}$$

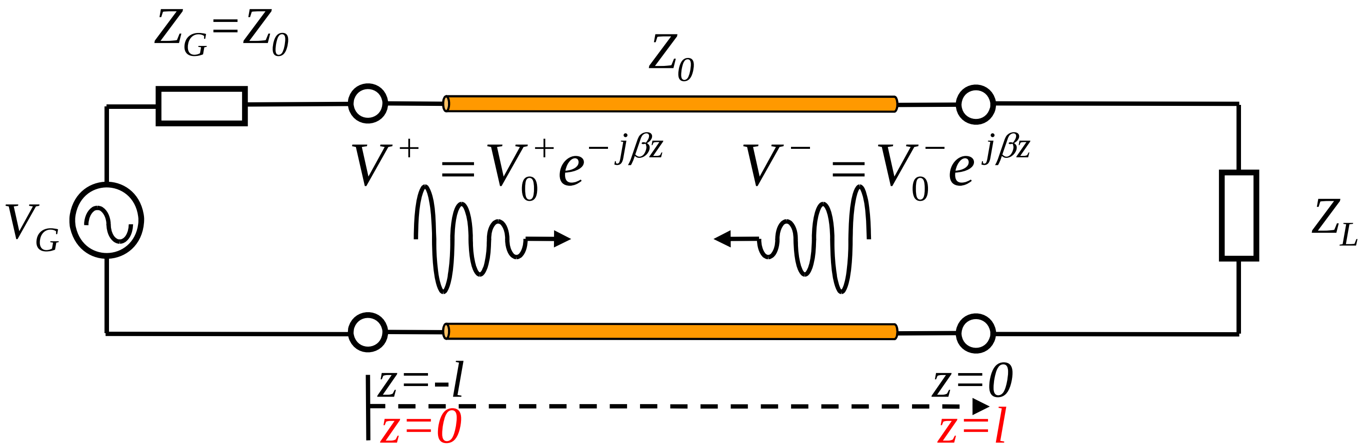
- Because **at the load** ($z=0$) the impedance of the line is Z_L:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0 \quad \Rightarrow \quad \rho_L = \rho(0) = \frac{Z_L - Z_0}{Z_L + Z_0}$$

voltage reflection coefficient at load

- One has to be careful with the coordinate axis chosen to define the reflection coeficient along the line.
  - by definition:

$$\rho(z)=\frac{V^-(z)}{V^+(z)}$$



  - coefficient at load: $\rho_L=\rho(0)=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_0}{Z_L+Z_0}$ $\quad$ $\rho_L=\rho(l)=\frac{V_0^-}{V_0^+}e^{j2\beta l}=\frac{Z_L-Z_0}{Z_L+Z_0}$
  - coefficient at input port: $\rho_{IN}=\rho(-l)=\frac{V_0^-}{V_0^+}e^{-j2\beta l}$ $\quad$ $\rho_{IN}=\rho(0)=\frac{V_0^-}{V_0^+}$
  - in both cases: $\rho_{IN}=\rho_L e^{-j2\beta l}$
  - however: $\rho(z)=\rho_L e^{j2\beta z}$ $\quad$ $\rho(z)=\rho_L e^{-j2\beta(l-z)}$

- Because **at the input port of the line** (z=-l) the impedance is:

$$\rho_{IN} = \rho(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \rho(0)\, e^{-2j\beta l} = \rho_L\, e^{-2j\beta l}$$

$$\rho_{IN} = \rho_L\, e^{-2j\beta l}$$

voltage reflection coefficient at input port

- At any point in the line: $\rho(z) = \rho_L\, e^{2j\beta z}$

- Reflection coefficient is a complex number.
- For a passive load the magnitude of the reflection coefficient is always lower than 1.

$$0 \le |\rho| \le 1$$

Example: Reflection coefficient of a loaded lossy transmission line. Taking into consideration the equations of the voltage and current waves flowing in a lossy transmission line find the equation for the input impedance on the line.

$$\rho_{IN} = \rho(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-\gamma l}}{V_0^+ e^{\gamma l}} = \rho_L e^{-2\alpha l} e^{-2j\beta l}$$

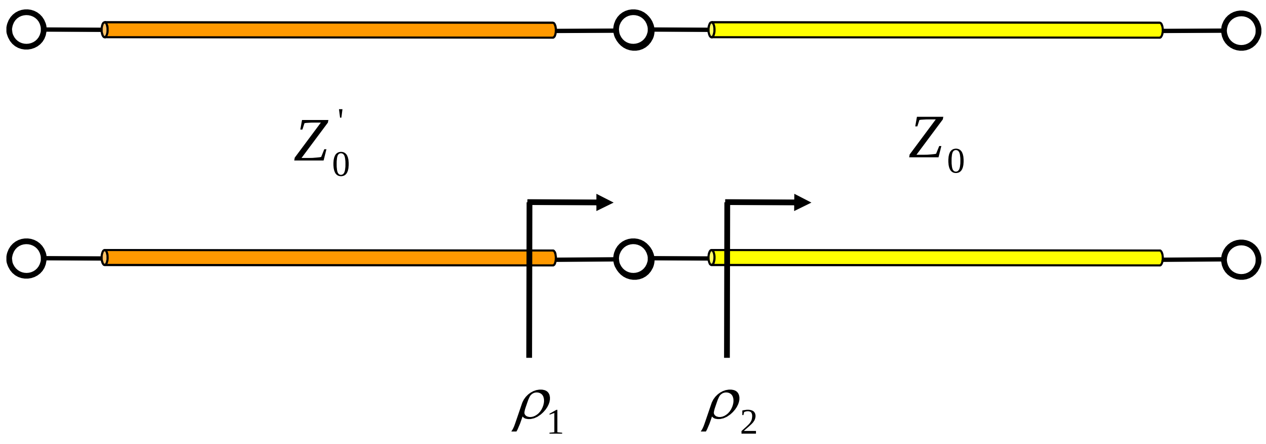
The reflection coefficient is attenuated when the line increases its length.

Take your time…

Standard loads. Which is the reflection coefficient corresponding to, respectively, an open circuit, short circuit, and reference impedance (Z_0)?

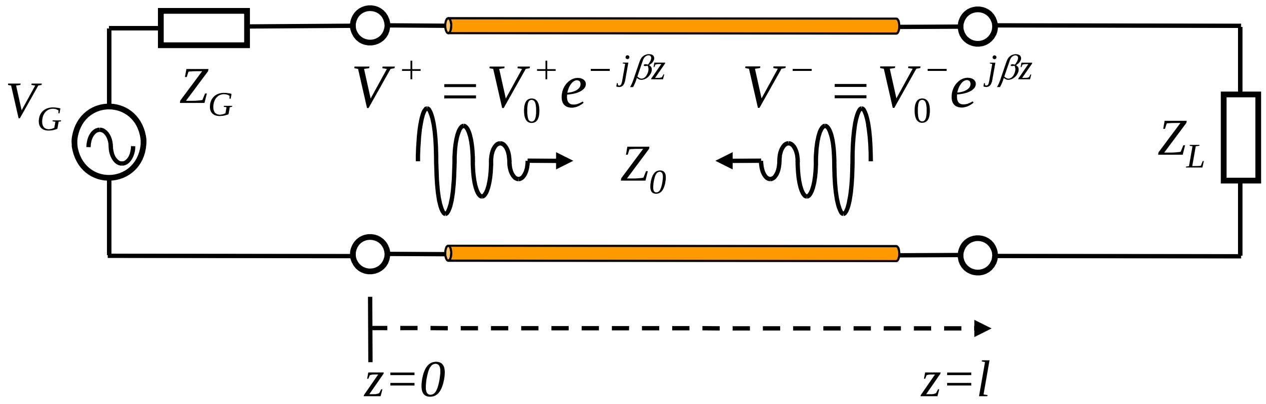
$$\rho_L\big|_{Z_L\to\infty} \qquad \rho_L\big|_{Z_L=0} \qquad \rho_L\big|_{Z_L=Z_0}$$

Line transition. Is the voltage reflection coefficient the same at both sides of the transitions between the transmission lines?



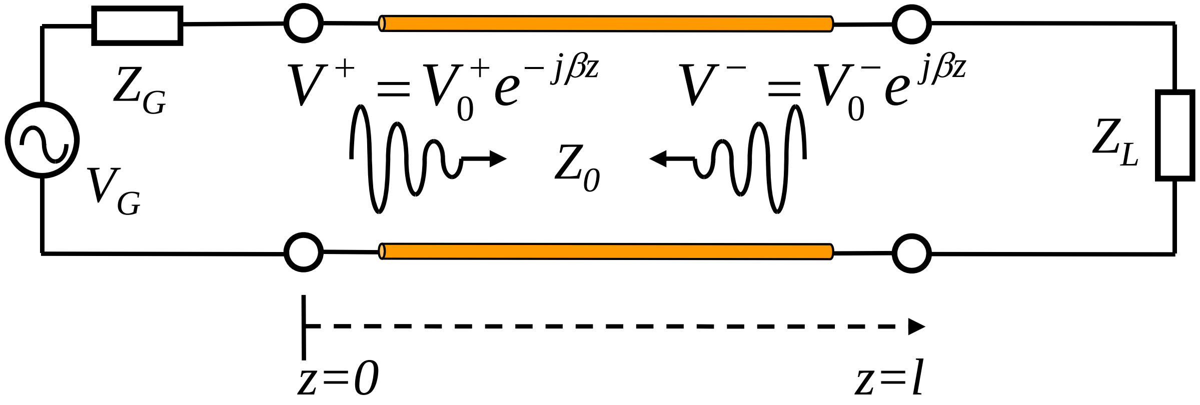
Take your time…

Mismatched lossless transmission line. By appliying boundary conditions to the ports of the lossless transmission line of the figure, find the voltage reflection coefficients ρ_{IN} and ρ_L, and the magnitude of the progressive wave V_0^+.



Take your time…

Solution. Mismatched lossless transmission line.



- at the output port of the line:
  - voltage:

$$V^+(l)+V^-(l)=V_0^+e^{-j\beta l}+V_0^-e^{j\beta l}=I(0)Z_L=\frac{Z_L}{Z_0}\left[V_0^+e^{-j\beta l}-V_0^-e^{j\beta l}\right]$$

  - defining the load reflection coefficient ρ_L…

$$\rho_L=\frac{V^-(l)}{V^+(l)}=\frac{V_0^-}{V_0^+}e^{2j\beta l}$$

  - … we get:

$$\rho_L=\frac{Z_L-Z_0}{Z_L+Z_0}$$

- and at the input port:
  - voltage and current:

$$\left.\begin{aligned}&V^+(0)+V^-(0)=V_G-I(0)Z_G\\&\frac{V^+(0)}{Z_0}-\frac{V^-(0)}{Z_0}=I(0)\end{aligned}\right\}$$

  - defining ρ_{IN}…

$$\rho_{IN}=\frac{V^-(0)}{V^+(0)}=\rho_L e^{-j2\beta l}$$

  - and ρ_G…

$$\rho_G=\frac{Z_G-Z_0}{Z_G+Z_0}$$

  - …we get:

$$V_0^+=V_G\frac{Z_0}{Z_0+Z_G}\frac{1}{1-\rho_{IN}\rho_G}$$

- The time-average power flow along the line at point z is:

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(z)I^*(z)\right] = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^-\right|^2}{2Z_0} = P^+ - P^-$$

$$P_{av} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\rho(z)\right|^2\right)$$

- Incident power to the line:

$$P_{inc} = \frac{\left|V_0^+\right|^2}{2Z_0}$$

- Reflected power at the load:

$$P_{ref} = \frac{\left|V_0^+\right|^2}{2Z_0}\left|\rho_L\right|^2$$

- To avoid the existence of reflected waves ($\rho=0$) on a loaded transmission line the load impedance Z_L should be equal to the characteristic impedance Z_0. Such a load is said to be a **matched load**.

- Some considerations derive from the previous equations:

power on the load

$$Z_L = Z_0 \quad \Rightarrow \quad P^- = 0 \quad P_L = P^+$$

$$Z_L = \{0, \infty\} \quad \Rightarrow \quad P^- = P^+ \quad P_L = 0$$

- The power carried by the positive flowing wave can be greater that the average power flowing on the line

- When the load is mismatched not all the available power is the delivered to the load. This loss is called **return loss** (RL):

$$RL = -20 \cdot \log|\rho_L| \qquad [\mathrm{dB}]$$

- What happens in case of a **lossy** transmission line?

For a forward propagating wave in a transmission line with length l:

$$V(z) = V_0^+ e^{-\gamma z}$$

$$I(z) = \frac{1}{Z_0} V_0^+ e^{-\gamma z}$$

$$P^+(l) = \frac{1}{2}\mathrm{Re}\left[V \cdot I^*\right] = P^+(0)\, e^{-2\alpha l}$$

attenuation

incoming power at input port

Wave attenuation (L) between planes z=0 and z=l:

- in decibels [dB]: $L\,[\mathrm{dB}] = 10\log\left(\frac{P^+(0)}{P^+(l)}\right) = \alpha l\ 20\log e = 8.686\,\alpha l$

Take your time…

Feeding and antenna. A 50-Ω antenna is fed with a signal of 100 W by means of a 75 Ω cable. Find the power finally arriving to the antenna.

a) 4 W b) 20 W c) 80 W d) 96 W

- The voltage on each point of a line depends on the load attached at its end:

$$\left|V(z)\right| = \left|V_0^+\right| \left|1+\rho_L e^{j2\beta z}\right|$$

  - When the transmission line is matched, the magnitude of the voltage on the line is constant:

$$\left|V(z)\right| = \left|V_0^+\right|$$

  - When the transmission line is not matched the overlap of an incoming and a reflected wave leads to a **standing wave** whose magnitude oscillates with the position on the line:

$$V_{\max} = \left|V_0^+\right|\left(1+\left|\rho_L\right|\right)$$

$$V_{\min} = \left|V_0^+\right|\left(1-\left|\rho_L\right|\right)$$

- A measure of the line mismatch is the **Voltage Standing Wave Ratio** (VSWR):

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+\left|\rho_L\right|}{1-\left|\rho_L\right|}$$

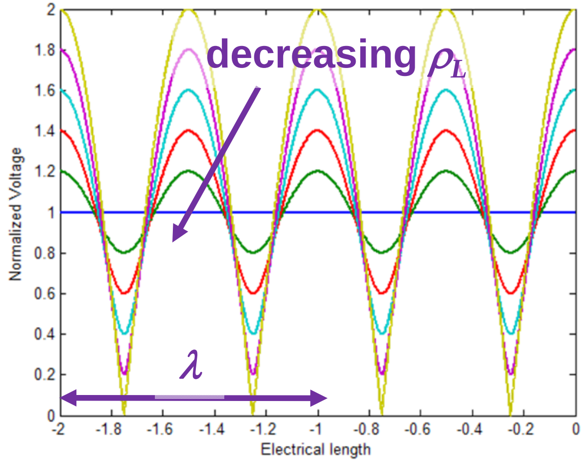
- SWR is a real positive number: $1 \le VSWR \le \infty$

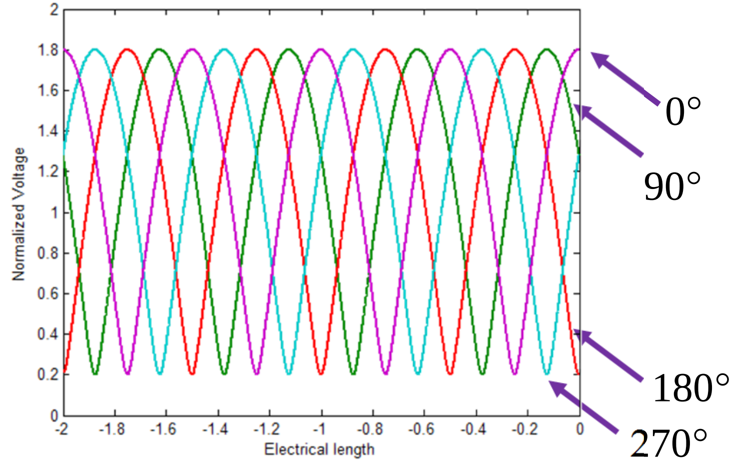
matched load $\Rightarrow$ $\rho_L = 0$ $\Rightarrow$ $VSWR = 1$

total reflection $\Rightarrow$ $\left|\rho_L\right| = 1$ $\Rightarrow$ $VSWR = \infty$

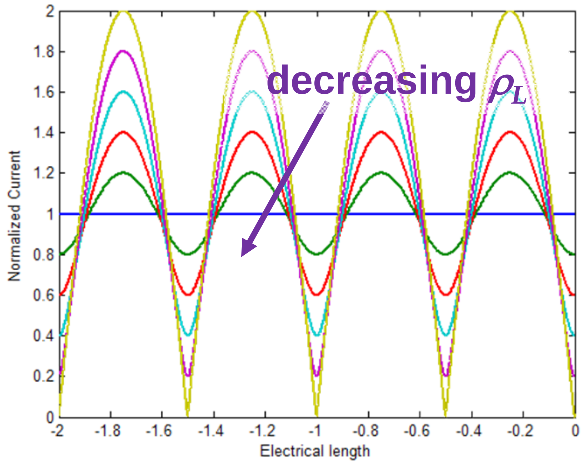
Example: Voltage on a transmission line. Plot the normalized voltage and current on a transmission line 2λ in length as a function of position on the line (z). Consider the following loads: $\rho_L=\{0,0.2,0.4,0.6,0.8,1\}$ and $\rho_L=\{0.8,\ 0.8\,e^{j90°},\ 0.8\,e^{j180°},\ 0.8\,e^{j270°}\}$.

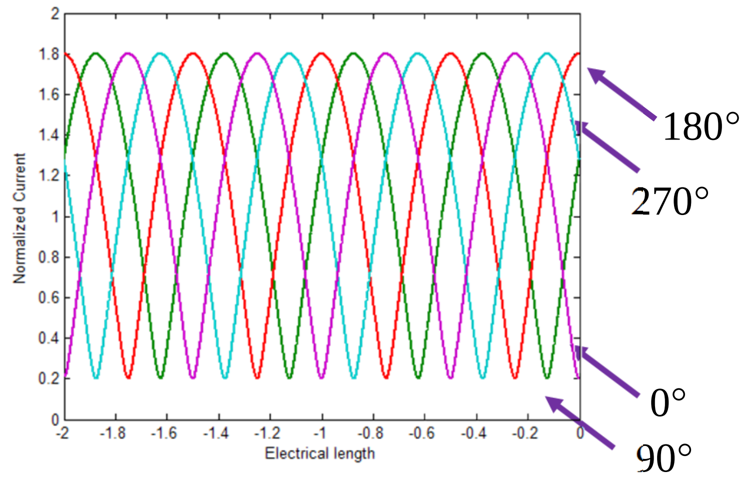
$$\frac{|V(z)|}{|V_0^+|}=\left|1+\rho_L e^{j2\beta z}\right|$$





$$\frac{|I(z)|}{|I_0^+|}=\left|1-\rho_L e^{j2\beta z}\right|$$



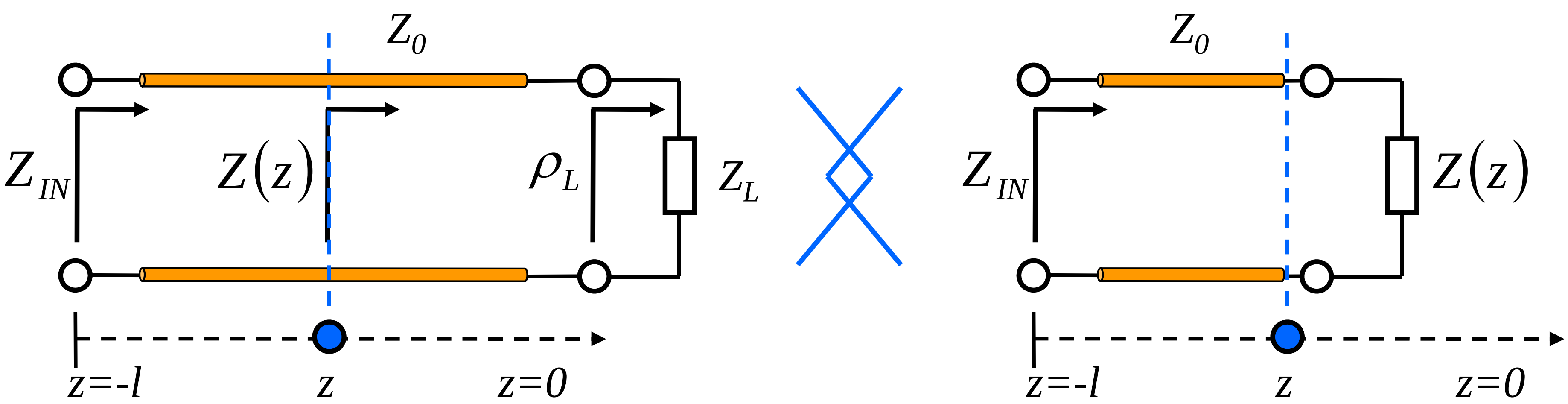


- Line impedance varies with position (z):

$$Z(z)=\frac{V(z)}{I(z)}=Z_0\frac{V_0^+\,e^{-j\beta z}+V_0^-\,e^{j\beta z}}{V_0^+\,e^{-j\beta z}-V_0^-\,e^{j\beta z}}=Z_0\frac{1+\rho_L e^{j2\beta z}}{1-\rho_L\,e^{j2\beta z}}$$

- The input impedance of a line loaded with Z_L is:

$$Z_{IN}=\frac{V(-l)}{I(-l)}=Z_0\frac{Z_L\cos(\beta l)+jZ_0\sin(\beta l)}{Z_0\cos(\beta l)+jZ_L\sin(\beta l)}=Z_0\frac{Z_L+jZ_0\tan(\beta l)}{Z_0+jZ_L\tan(\beta l)}$$



- The case of highly reflective loads:

short-circuited stub: $Z_{IN} = jZ_0 \tan(\beta l)$ ⇒ **inductive impedance**

open-circuited stub: $Z_{IN} = \dfrac{Z_0}{j\tan(\beta l)}$ ⇒ **capacitive impedance**

- Quarter-wave long transmission lines perform as **quarter-wave (impedance) transformers** or **impedance inverters**, being the input impedance inversely proportional to the load impedance.

When: $l = \dfrac{\lambda}{4} + n\dfrac{\lambda}{2} \quad n = 0,1,2,\ldots$ ⇒ $Z_{IN} = \dfrac{Z_0^2}{Z_L}$

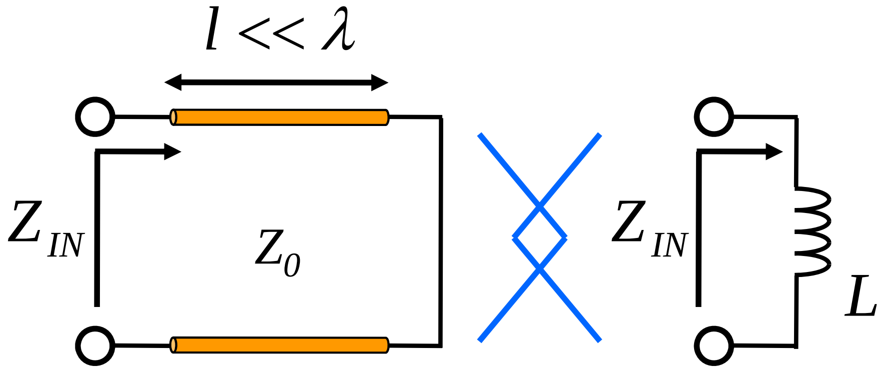
- Otherwise, any line whose length is any multiple of λ/2 does not transform the load impedance, regardless of the characteristic impedance.

When: $l = n\dfrac{\lambda}{2} \quad n = 1,2,3,\ldots$ ⇒ $Z_{IN} = Z_L$

Example: Emulating lumped elements with electrically-short transmission lines.
Suggest what lumped elements can be simulated by means of short transmission lines when these line end, respectively, with an open circuit and with a short circuit.

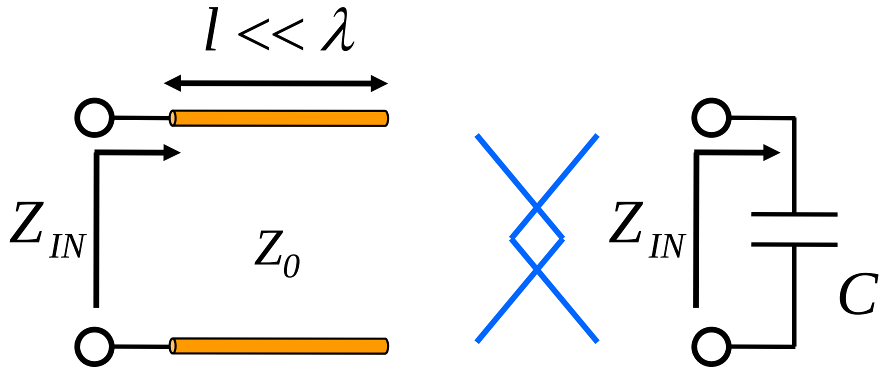
short-circuited short-stub:

$$Z_{IN} \approx jZ_0\beta l = j\omega L \quad \text{being: } L = \frac{Z_0 l}{v_p}$$

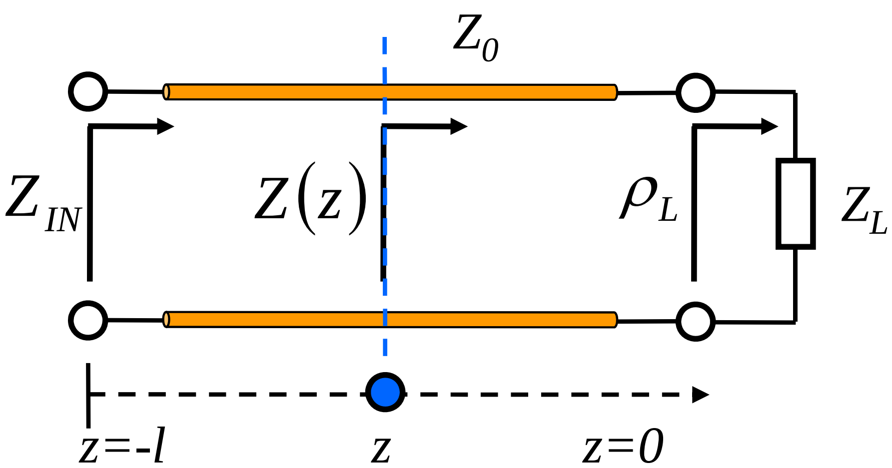


open-circuited short-stub:

$$Z_{IN} \approx \frac{Z_0}{j\beta l} = \frac{1}{j\omega C} \quad \text{being: } C = \frac{v_p l}{Z_0}$$



Example: Input impedance of a loaded lossy transmission line. Taking into consideration the equations of the voltage and current waves flowing in a lossy transmission line, find the equation for the input impedance on the line.



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right)$$

at z=-l

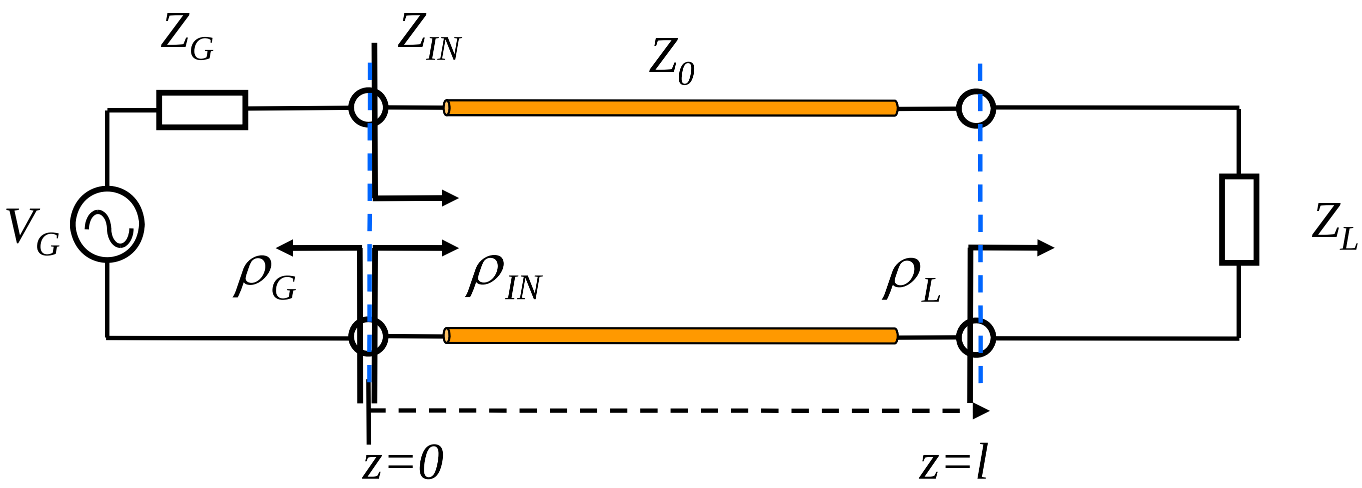
$$Z_{IN} = Z_0 \frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_0 \cosh(\gamma l) + Z_L \sinh(\gamma l)}$$

that is, also

$$Z_{IN} = Z_0 \tanh(\gamma l + \delta_Z)$$

$$\delta_Z = \tanh^{-1}\left(\frac{Z_L}{Z_0}\right)$$

- The voltage on the input port of the line can be calculated from the source voltage and depends on the load impedance.



$$V_0^+ = V_G \frac{Z_0}{Z_0 + Z_G} \frac{1}{1 - \rho_{IN} \rho_G}$$ being: $$\rho_G = \frac{Z_G - Z_0}{Z_G + Z_0}$$

at z=0: $$V(0) = V_G \frac{Z_{IN}}{Z_{IN} + Z_G} = V_0^+ + V_0^- = V_0^+ (1 + \rho_{IN})$$

and being: $$Z_{IN} = \frac{V(0)}{I(0)} = Z_0 \frac{1 + \rho_{IN}}{1 - \rho_{IN}}$$

- Consequently:

when: $Z_G = Z_0$ ⇒ $V_0^+ = \dfrac{V_G}{2}$

$Z_{IN} = Z_0$ ⇒ $V_0^+ = V_G \dfrac{Z_0}{Z_0 + Z_G}$

the power delivered from the source to the line (and if it has no losses, delivered to the load):

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(0)I^*(0)\right] = \frac{|V_G|^2}{2}\left|\frac{Z_{IN}}{Z_{IN}+Z_G}\right|^2 \mathrm{Re}\{Z_{IN}\}$$

being: $Z_{IN} = R_{IN} + jX_{IN}$

and: $Z_G = R_G + jX_G$

$$P_{av} = \frac{|V_G|^2}{2}\frac{R_{IN}}{(R_{IN}+R_G)^2 + (X_{IN}+X_G)^2}$$

Take your time…

Power delivered to the load. Consider a lossless transmission line connected to a fixed source impedance $Z_G=R_G+jX_G$. Find the power delivered to the load (or the power delivered to the transmission line) in the following two cases: a) when the load is matched to the line ($Z_L=Z_0$); and b) when the generator is matched to the loaded line ($Z_G=Z_{IN}$).

Take your time…

Solution: Power delivered to the load.

- Load matched to the line (Z_L=Z_0).

$$P_{av} = \frac{|V_G|^2}{2} \frac{Z_0}{(Z_0 + R_G)^2 + X_G^2}$$

- Generator matched to the loaded line (Z_G=Z_{IN}).

$$P_{av} = \frac{|V_G|^2}{8} \frac{R_G}{R_G^2 + X_G^2}$$

Take your time…

Impedance for maximum power transfer or available power. Assuming that the generator series impedance is fixed find the input impedance Z_{IN} to achieve the maximum power transfer to the load (lossless transmission line). In that case, find the power delivered to the load.

Take your time…

Solution: Impedance for maximum power transfer.

- Maximizing the power transfer…

$$\left.\begin{array}{l}\dfrac{\partial P_{av}}{\partial R_{IN}}=0\\[2ex]\dfrac{\partial P_{av}}{\partial X_{IN}}=0\end{array}\right\}\Rightarrow\left.\begin{array}{l}R_G^2-R_{IN}^2+\left(X_G+X_{IN}\right)^2=0\\R_{IN}\left(X_G+X_{IN}\right)=0\end{array}\right\}\Rightarrow\left.\begin{array}{l}R_{IN}=R_G\\X_{IN}=-X_G\end{array}\right\}\Rightarrow Z_{IN}=Z_G^*$$

… we get that the input impedance of the line should be the complex conjugate of the source impedance. This condition is called **conjugate matching**.

- In this case, the power transferred to the load is (lossless transmission line):

$$P_{av}=\frac{\left|V_G\right|^2}{8R_G}$$